Remarks on optimization and the prediction of bone adaptation to altered loading

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The topic considered is the close connection between computational bone adaptation –to mechanical loading and structural optimization. An open mathematical problem connecting computational bone adaptation based on local surface adaptation rules and global structural-optimization methods is described.

Bone adaptation to mechanical loading

The remodeling processes in living bone are the mechanisms by which bone tissue adapts its overall structure to changes in its mechanical load environment. The time scale of the remodeling processes is on the order of months or years. Changes in lifestyles that alter the loading environment, for example taking up jogging, have remodeling times on the order of many months. The concern is with adaptation of the bone structure that occurs in the individual rather that the adaptations of the species. In that sense, the viewpoint here is not Darwinian. The rationale for computational modeling of the process stems principally from the fact that the problem of the long-term stability (ten years and longer) of structural bone implants, *e.g.*, the total hip, has not been satisfactorily solved. It also stems from the search for strategies to overcome osteoporosis as well as the fact that exposure to microgravity tends to induce bone loss. Recent books that review aspects of bone remodeling include Frost (1986), Martin et al. (1998), Cowin (2001) and Currey (2002).

Traditionally, animal experiments have been employed to investigate bone response to implants. Animal experiments are, however, prohibitively expensive for addressing the question of long-term stability. Computer modeling is reliable, inexpensive and fast. Although computer models cannot take the place of animal experiments, they have the potential to reduce the number of animal experiments by focusing experimental design.

The objective of computational modeling of the stress adaptation process is closely associated with the design of bone implants. The ultimate objective is the development of a computational procedure for predicting shape changes in whole bones as well as the adaptation of the spongy bone tissue architecture to the insertion of an implant. This ideal computational procedure would permit the implant designer to see, almost instantaneously, the predicted effects of the implant on the patient's bone after years in the body, and thus speed the design process. Rapid progress is being made toward this goal (cf., e.g., Hart, 2001).

Structural optimization

There is a close connection between the objectives of computational bone remodeling and the field of mechanics known as structural optimization (Petersen and Bendsøe, 1999; Bendsøe, 1995). *Structural optimization* is a term used to cover a number of optimization techniques associated with structural mechanics; it includes shape, size, and topological optimization. It also includes material optimization, which overlaps strongly with material design. *Shape optimization* is the term used to describe the process of finding the optimum shape of a domain, the shape being the design variable; see Haftka and Grandhi (1986). *Size optimization* is the term used to describe the process of finding the optimum dimension of a structural shape, say, the thickness of a plate. *Topology optimization* is a method that applies equally well to determining the connectivity of a structural domain or to material optimization (design). For a survey of structural optimization, see Bendsøe (1995) and for applications Petersen and Bendsøe (1999).

Surface bone remodeling bears a great deal of similarity to shape optimization because surface bone remodeling has to do with changes in the overall shape of a single bone. Internal bone remodeling has many characteristics and techniques in common with topology optimization. One application of topology optimization is to fashion the shape of a truss from an amorphous structural domain in a manner similar to a sculptor fashioning a sculptural art form from an amorphous marble domain. Internal bone remodeling is used in a manner similar to topology optimization to fashion local trabecular architecture. Trabecular architecture is treated as both a structure and a material in mechanics. It is treated by some as a structure, by others as a material and, by still others, both ways. Similar remarks may be made about topology optimization because it is applied to both structures and to material optimization.

On the use of the word "optimum" in physical sciences and biology

Biologists and physical scientists mean something different when they use the word "optimum." The physical scientist means the mathematical determination of the extrema of a particular function of one or more variables with respect to a subset of the variables, sometimes under specified constraints. The meaning the biologists assign to the word "optimum" is less specific and generally means that they cannot imagine that nature could have accomplished this biological result in a better way. Some German speaking young people use the word "optima" to mean "wonderful;" many biologists appear to use "optimum" in the same way. As Humpty Dumpty said to Alice "When I use a word it means just what I choose it to mean – neither more nor less." The observation of this difference in meaning associated with "optimum" in the different academic areas is made so that those on both sides can understand the meaning of the word in the other discipline.

An important open mathematical question concerning global optimization models and phenomenological local models

The words global and local are used to distinguish the events associated with the entire structure from those that occur at specified parts of the structure. In the case of bones, the whole bone, or an entire bone-implant system is the global system and a local surface rule for the deposition or removal of bone mass at a boundary point is the local rule. The local model is used to determine the local remodeling and the local surface movement predicted. Global consistency (but not necessarily optimization) is achieved because the global structure is an elastic object and the surface movement rule is stress or strain dependent. The elliptic character of elastostatics provides the global consistency for the surface movement rule models. The important open mathematical question concerning global optimization models and phenomenological local models is why do the two apparently distinct approaches give the same results. There are numerous examples (Petersen and Bendsøe, 1999) of global optimization problems and local surface adaptation rules results that are equivalent.

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