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At the University of Florida, we apply optimization to biomechanics problems in three ways: (1) to explain benefits of bone composition to structural performance, (2) to produce biomimetic structural designs, and (3) to facilitate modeling of human movement and muscle actions.

Bone Composition and Structural Performance

Bones constitute a substantial fraction of skeletal mass in humans and larger mammals, and so minimizing the weight of bones appears to be important biologically. The rapid loss of bone mass that astronauts experience in space confirms that bone mass is expensive for the body to maintain. This indicates that structural optimization should be useful for explaining at least some of the features encountered in bone structures.

While it is well known that bones will adapt and respond to mechanical loading for structural considerations, it is not clear that they can attain optimality. This is due to the fact that bones may be able to use only remodeling strategies that respond to local conditions, such as stress or strain, rather than to the overall state of stress and strain. Therefore, there has been much work, dating back to Wolff's law, that posits local remodeling laws in order to explain observed patterns of bone composition and architecture. Much of this work has dealt with length scales on the order of whole bones and visible features within them.

We have chosen to focus on microstructural details, in particular, near a type of natural hole found in bones. Major blood vessels penetrate the thick outer shell of most bones through such a hole called the nutrient foramen. We hypothesized that bone composition in their vicinity mitigated potential stress concentrations associated with holes, as foramina are rarely implicated in bone fracture. We studied the 2 mm diameter foramen in an equine third metacarpus (**Figure 1**). We found an interesting pattern of mineralization and porosity that created a compliant region near the foramen and stiffer regions at a distance of about two diameters away from it (**Figure 2**).

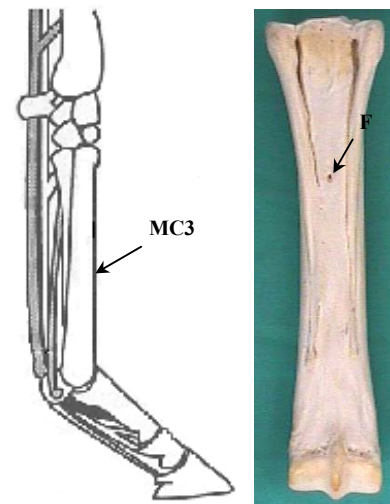


Figure 1. Lateral view (left) of the equine forelimb and the location of the third metacarpus (MC3). Palmar view (right) of the MC3 and the nutrient foramen (F).

We then sought to perform structural optimization of the bone in order to check whether the observed patterns were optimal. We had to combine relationships found in the literature between composition on one hand (our observed quantities) and elastic and yield properties on the other. We also postulated that the objective function of the bone was to minimize a failure index

(loosely, the ratio of local strength to stress) in the vicinity of the hole. Even though we used general optimization algorithms rather than local remodeling, we were able to reproduce the patterns of stiffness distribution that we found in the bone (**Figure 2**). This increased our confidence in our modeling of the bone and in the objective function that we chose.

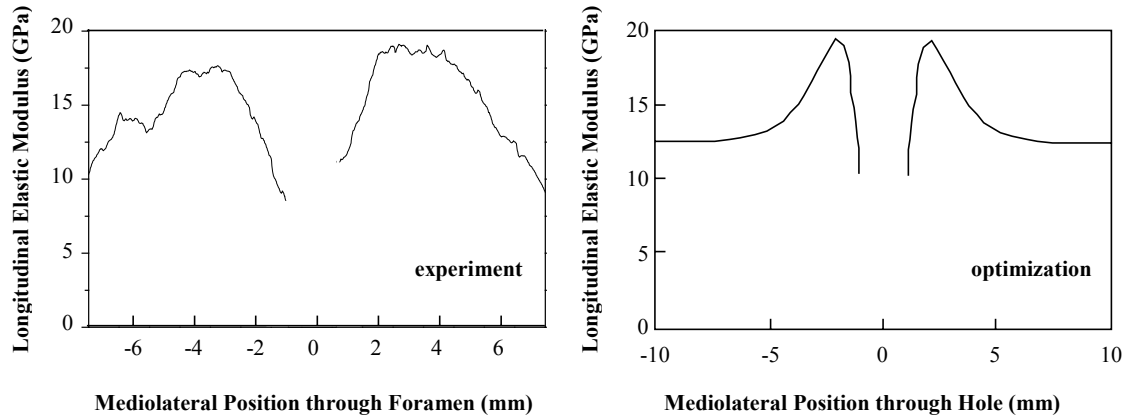


Figure 2. Experimentally derived (left) longitudinal elastic modulus along a line through the foramen. Optimization derived (right) modulus along a line through a circular hole in a plate.

We intend to pursue composition optimality about smaller discontinuities in bone. For example, osteons are repeating cylindrical units (~200 μm diameter) comprising the adult cortical bone of many mammals (**Figure 3**), and are akin to fibers in structural composites. Each osteon is comprised of concentric lamellae (~10 μm thick) that surround a cylindrical channel, the Haversian canal (~50 μm diameter). Interspersed throughout the osteon between some lamellae are spheroidal lacunae (~10 μm), in which reside the sensory osteocyte cells of bone. Emanating from each lacunae are channels, the canaliculi (~0.5 μm), through which the osteocytes send processes to form the sensory network. We hope to see if on these smaller scales structural considerations dictate optimal compositions or if other biological considerations dominate.

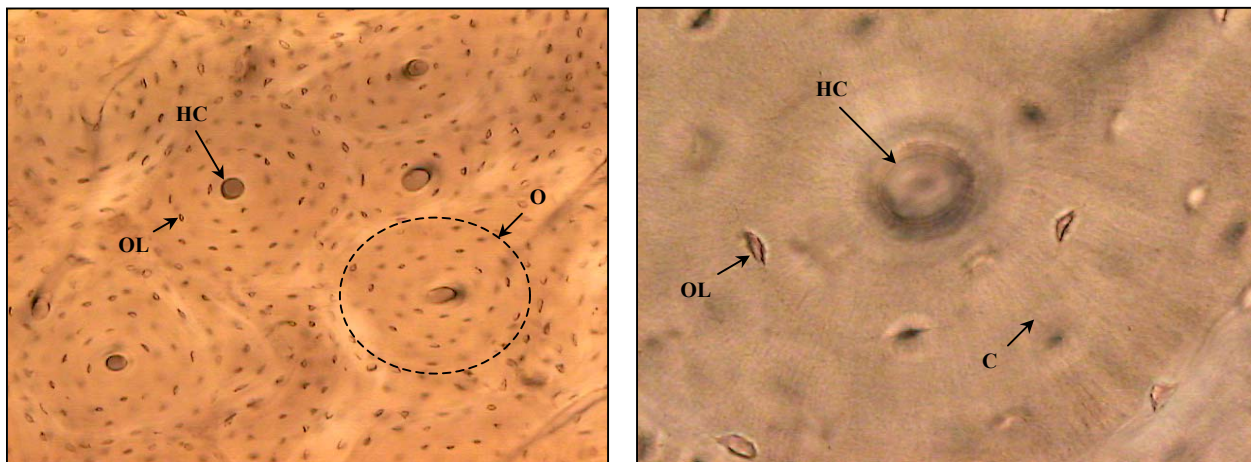


Figure 3. Photomicrograph at left of a thin transverse section through a bovine metacarpus. Osteons (O) are the slightly darker circular features with a Haversian canal (HC) at their centers. Osteocyte lacunae (OL) are the small dots. Enlarged view of a portion of a single osteon at right. The canaliculi (C) are the myriad of tiny “cracks” emanating from each lacunae and the Haversian canal.

Biomimetic Structural Designs

Optimized structures found in nature can be sometimes imitated in engineering structures. The recent interest in functionally graded materials make bone structures interesting because bones are naturally functionally graded. We set out to investigate whether the optimal structural pattern near the equine foramen would lead to useful insights for the design of functionally graded materials. In particular, we were interested in functionally graded foams, where density is easy to control.

We isolated the variation in apparent density ρ about the foramen as worthy of mimicking in designs because of its relationship with elastic modulus E and yield strength σ_Y for an important class of materials including structural foams (plastic and metal) and bone. These materials can be described as “power law materials” because E and σ_Y are power law functions of ρ , such that $E \sim \rho^b$ and $\sigma_Y \sim \rho^\beta$. When the failure index based optimization procedure is used, the failure index can be written as a function of the material exponent β/b . This material exponent, which is different for different materials, then strongly influences the resulting optimum design.

We constructed a model of a plate containing a central circular hole and allowed the apparent density to vary radially to preserve axisymmetry about the hole. The material, although inhomogeneous, was everywhere isotropic. We produced a series of designs for different materials exponents (Figure 4). Again, we found remarkable similarity between our optimal design and the foramen for $\beta/b = 0.5$, which agrees with published values for cortical bone. Other designs were produced for other material exponents. For example, at some point (below $\beta/b \approx 0.75$) when strength does not increase fast enough with density compared to modulus, it “pays” to divert stresses from the hole, as the foramen does. We were particularly interested in the design curve of one material because we could fabricate a real plate out of this material.

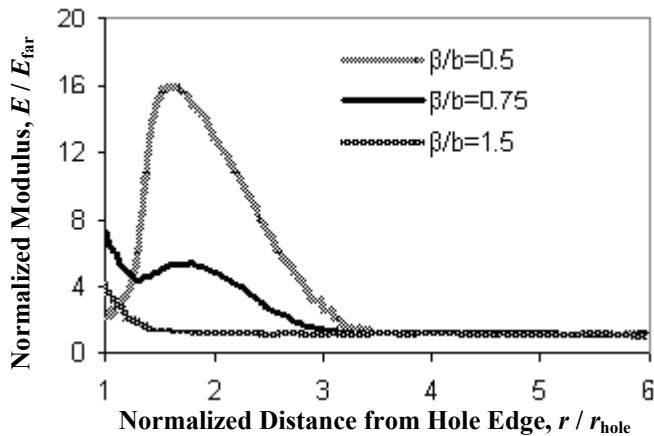


Figure 4. Optimal distribution of normalized modulus around a hole in a plate model for different material exponents. Local modulus and strength are power law functions of density, and β and b are the exponents. These distributions result from minimizing the failure index. For bone $\beta/b \approx 0.5$.

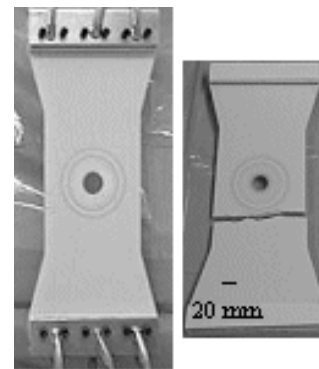


Figure 5. Biomimetic foam plate, with rings that approximate modulus distribution for $\beta/b = 0.84$.

From previous experience, we knew of a power law material – polyurethane foam – that is readily available and relatively easy to incorporate into a biomimetic design. We supplied such a design to a manufacturer of orthopedic models, who has in stock a class of polyurethane foams for which $\beta/b = 0.84$. Our design was a “dog bone” plate containing a circular hole, about which a series of discrete rings of different density were incorporated (**Figure 5**). Functionally grading manufacturing technologies are emerging, and this discrete (in contrast to continuous) gradation represented the best method in the short term. Even with this design, our model predictions under uniaxial tension indicated a reduction in the failure index from 3.1 for the homogeneous design to 1.1 for our biomimetic design. Our predictions were in good agreement with experimental results, even the location of the failure, “downstream” of the hole and not to the side of it. Our biomimetic plate was approximately twice the strength of the homogeneous plate (or, for the same strength, half the weight).

Modeling of Human Movement and Muscle Actions

“Virtual human” models can be used to optimize orthopedic surgical parameters, rehabilitation strategies, orthopedic implant designs, or human-machine interfaces. To be effective, such models must be tuned to match the structure and parameters of the patient or subject. Furthermore, the models must simulate the musculoskeletal system under in vivo functional conditions. This implies that dynamic rather than static modeling approaches are needed to perform functional simulations, similar to the functional simulations of commercial products performed in industry for design purposes.

Optimization plays an essential role in our development of subject-specific dynamic musculoskeletal models. Below is a brief description of each of the primary areas involving optimization:

- (1) Determination of patient-specific joint axes for kinematic models. These models form the skeletal structure for dynamic musculoskeletal models.
- (2) Determination of patient-specific mass and inertia properties for dynamic models. These parameters are also required for predictive dynamic musculoskeletal models.
- (3) Determination of joint control force and torques for dynamic simulations that match experimental motion and ground reaction force measurements. The ability to match experimental movement data provides initial confidence in the subject-specific model structure and parameters.
- (4) Prediction of muscle forces (an indeterminate problem) that produce novel movements for which no experimental data are available. Knowledge of muscle forces is needed to mimic the function of the human controller as well as to determine joint contact pressures influenced by muscle co-contractions (**Figure 6**).
- (5) Prediction of material parameters in elastic contact models of human joints. By tuning materials parameters so that joint contact simulations match experimental measurements, the

simulations can be used to predict joint function under other conditions for which experimental data are not available.

The simulations used for some of these applications require as much as 10 to 15 minutes of CPU time, making traditional single-processor optimizations unrealistic. For this reason, we have developed parallel processing optimization methods involving a particle swarm global optimization algorithm as well as a more traditional gradient-based algorithm. Other simulations require only a few seconds of CPU time, making built-in optimization functionality available in commercial simulation programs sufficient for the task. In all applications, specification of an appropriate optimization problem formulation is one of the most important steps in the solution process.

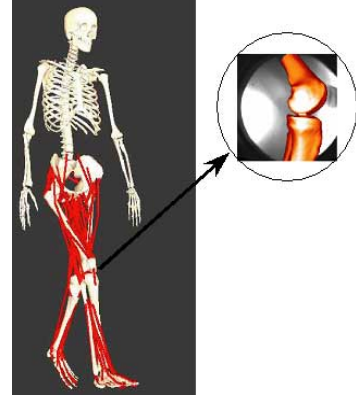


Figure 6. In order to predict contact conditions in the knee joint, muscle forces during walking and other activities must be predicted by models. These models typically minimize an objective function such as the muscle stresses cubed.

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